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This final report on grant AFOSR 84-0159 describes research accomplished during the period July 1, 1984 - June 30, 1988 by Principal Investigator Francisco J. Samaniego and his collaborators (Boyles, Reneau and Whitaker) under grant support. Results in four broad areas are described: (1) Statistical Inference for Repairable Systems, (2) Inference Based on Observed Extreme Values, (3) Inference for Nonparametric Reliability Models and (4) Structural Results in Reliability. Keywords: Mathematical models. (AW)

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Reliability Modeling and Inference for Coherent Systesms Subject to Aging, Shock or Repair

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by

Francisco J. Samaniego, Principal Investigator

June 30, 1988

The work accomplished by the Principal Investigator under grant AFOSR 84-0159 from its initiation on July 1, 1984 through the present may be classified into four broad areas, each of which is discussed in turn.

A. Statistical Inference for Repairable Systems.

In Whitaker and Samaniego [16], we focus on the development of estimation procedures for the life distribution F of a new system based on data on system lifetimes between consecutive repairs. The 'Imperfect Repair' model introduced by Brown and Proschan [5] postulates that, at failure, the system is repaired to a condition as good as new with probability p and is otherwise repaired to the condition just prior to failure. In treating issues of statistical inference for this model, we begin by pointing out the lack of identifiability of the pair (p,F) as an index of the distribution of interfailure times $T_1, T_2, \ldots$ We show that data pairs (T_i, Z_i), $i = 1,2,\ldots$ are necessary to render the parameter pair (p,F) identifiable, where Z_i is a Bernoulli varaiable which records the mode of repair (perfect or imperfect) following the i^{th} failure. We demonstrate that the nonparametric maximum likelihood estimator of $\hat{F}$ exists only in special cases, but that a neighborhood maximum likelihood estimator always exists and may be derived in closed form. We demonstrate the strong uniform consistency of $\hat{F}$ under mild assumptions, and prove weak convergence of an appropriately scaled version of $\hat{F}$ to a Gaussian process.

These results are shown to apply to various experimental designs (including renewal testing and inverse sampling) and to extend to the age-dependent imperfect repair model of Block, Borges and Savits [2].

Lindley and Singpurwalla [8] have studied the modeling of failure time data from experiments involving changing environments. They consider, for example, the situation in which two components are tested in a controlled environment and subsequently tested in a (perhaps harsher) operating environment. Letting Exp(λ) represent the exponential distribution with density

$$f(t) = \lambda e^{-\lambda t}, \quad t>0,$$

they analyze the model in which component i is assumed to be distributed according to Exp(λ_i) in laboratory experiments and according to Exp($\eta\lambda_i$) under operating conditions, where, typically, $\eta>1$. In Currit and Singpurwalla [6], this model is studied further, and the mechanics necessary for Bayesian estimation of λ (assuming $\lambda_1=\lambda_2=\lambda$) are developed. Samaniego [12] provides a complementary analysis, focusing on the maximum likelihood estimators of the parameters of the exponential distributions governing the lifetime of components observed under both laboratory and operating conditions. The treatment is general, applying to any number of components observed under dual conditions. Some potential applications of this analysis exist in modeling repairable systems. For example, if a collection of parallel systems in k non-identical components are tested until failure, and if these systems are repaired and retested, it seems reasonable to assume that the pre- and post-repair environments satisfy the Lindley-Singpurwalla assumptions.

B. Inference Based on Observed Extreme Values.

Samaniego and Whitaker [14] treat experiments in which only record-breaking values (e.g., values smaller than all previous ones) are observed. The data available may be represented as $X_1, K_1, X_2, K_2, \ldots$, where $X_1, X_2, \ldots$ are successive minima and $K_1, K_2, \ldots$ are the numbers of trials needed to obtain new records. The problem of estimating the mean of an underlying exponential distribution is considered, under both fixed sample size and inverse sampling schemes. Under inverse sampling, we demonstrate certain globl optimality properties of an estimator based on the "total time on tes" statistic. Under random sampling, it is shown than an analogous estimator is consistent, but can be improved for any fixed sample size.

When only a single sequence of random records are available, efficient estimation of the underlying distribution F is possible only in a parametric framework. However, under the assumption that the process of observing random records csan be replicated, Samaniego and Whitaker [15], derive and study the nonparametric maximum likelihood estimator $\hat{F}$ of F. The strong uniform consistency of this estimator is established (as the number of replications grows large), and its asymptotic distribution theory is described. The performance of $\hat{F}$ is compared to that of two reasonable competing estimators.

Boyles and Samaniego [3] have studied nonparametric estimation under nomination sampling, a sampling process in which every observation is the maximum of a random sample from some population. Assuming that all samples are taken from a single underlying distribution F, data may be viewed as consisting of pairs (X_i, K_i), where K_i is the size of the i^{th} sample and, given $K_i = k_i$, X_i is distributed according to F^{k_i}. Willemain

[18], who discussed nomination sampling in the context of health care delivery, proposed an estimator for the median of F under the assumption that $K_i = N$, a fixed integer. In our context, the assumption of a fixed sample size N from each population is relaxed; with K taken as random, the problem of nonparametric estimation of the distribution function F is considered. The nonparametric maximum likelihood estimator of F is the derived, its consistency is demonstrated, and its asymptotic behavior as a stochastaic proacess is identified. Conditions are given under which these asymptotic results hold for nonrandom K. A by-product of this development is the consistency of Willemain's estimator of the median. Several applications are considered. For example, nomination sampling arises naturally in certain reliability experiments; the applicability of derived estimator in problems involving designed redundancy is noted. A detailed analysis of data on 34 yearly maximum floods of the Nidd River is presented, and the estimator of the underlying flood distribution F is displayed. In Boyles and Samaniego [4], a "structural approach" to the estimation of F is considered, and the alternative method is shown to be especially well suited for semiparametric formulations of the problem, i.e., for situations in which K is appropriately modeled by a discrete parametric family of distributions, while F is unrestricted.

C. Inference for Nonparametric Reliability Models.

Much of the recent literature in statistical reliability theory focuses on nonparametric modeling and inference for life distributions. The nonparametric approach to modeling involves constructing classes of life distributions which reflect a particular qualitative aging behavior of components or systems. The appeal of these classes is that a few natural constraints suffice to capture intuitive notions of aging. On the other hand, statistical inference results have generally been more difficult to obtain for these models. We have been successful in treating a number of inference problems of this type. Three specific problems are described below.

(i) Let X be a positive random variable. The distribution F of X is said to be "new better than used in expectation," i.e., NBUE, if $E(X) \geq E(X-t|X>t)$ for all $t \geq 0$. Suppose $X_1, \ldots, X_n$, is a random sample from an NBUE distribution F. Whitaker and Samaniego [17] consider the problem of estimating F by a distribution which is itself NBUE. The estimator G_n, defined as the NBUE distribution supported on the sample which minimizes the (sup norm) distance between the NBUE class and the empirical distribution function, is studied. The strong uniform consistency of G_n is proven, and a numerical algorithm for obtaining G_n is given. ∎

(ii) Suppose the survival function S of a random variable X is a member of the NBU-$\{t_o\}$ class. By definition (see Hollander, Park, Proschan [7]), S must satisfy the following condition:

(1) $$S(x+t_o) \leq S(x)S(T_o) \text{ for all } x \geq 0.$$

Let $\{X_1, \ldots, X_n\}$ be a random sample from S. The goal is to find an estimator that itself belongs to the NBU-$\{t_o\}$ class and has the same asymptotic optimality properties as S_n.

Reneau and Samaniego [9] study the estimator $\hat{S}$ of S, given implicitly by

$$\hat{S}(x) = \begin{cases} S_n(x) & 0 \leq x \leq t_o \\ \min\left[S_n(x),\ \hat{S}(x-t_o)\hat{S}(t_o)\right] & x > t_o \end{cases}.$$

It is shown that $\hat{S}$ may be written in ter~ of the empirical survival curve S_n as follows:

$$\hat{S}(x) = \min_{0 \le k \le [x/t_o]} \left\{ S_n^k(t_o) \quad S_n(x-kt_o) \right\},$$

where [u] denotes the greastes integer less than or equal to u.

Reneau and Samaniego [9] establish the fact that the estimator $\hat{S}$ is a strong uniformly consistent NBU-$\{t_o\}$ estimator of S which converges to S at the best possible (pointwise and mean square) rates. Confidence procedures based on $\hat{S}$ are also discussed. ∎

(iii) For any set $A \subseteq [0,\infty)$, we define a survival function S to be New Better Than Used with respect to the set A (NBU-A) if

$$S(x|t) \le S(x|0) \quad \text{for all} \quad x \ge 0,\ t \in A,$$

where $S(x|t) = P(X>x+t|X>t)$.

This class clearly contains the NBU class of distributions, and it is in turn contained in the NBU-$\{t\}$ class for all $t \in A$. Reneau and Samaniego [10] study recursive estimation formulas for a number of special cases. When A is a finite set, the resulting estimators are shown to be strong uniformly consistent estimators of the corresponding survival curves. The same is true for the set $A = \{kt_o\colon k=1,2,\ldots\}$. When the set A contains a limit point, the methods used on the preceeding cases are not, in general, successful in producing a consistent estimator. Even in these cases, however, they do provide useful estimators in one important area of application, namely, when products are subjected to burn-in, with those items which fail before a predetermined time δ discarded. The survival function obtained by applying left-truncation to a survival function S,

$$S_\delta(x) = \begin{cases} 1 & \text{if } 0 \le x < \delta \\ S(x)\ /\ S(\delta) & \text{if } \delta \le x \end{cases}$$

is itself in the NBU-A class whenever S is. Thus, random sampling from a NBU-A distribution under fixed left-truncation such as burn- in yields a survival function of the desired type. When S has the property that $S(\delta)$

= 1 for some $\delta > 0$, the recursive approach to estimating S is shown to lead to a strong uniformly consistent estimator with optimal convergence rates. ∎

D. Structural Results in Reliability.

We have obtained a number of general results which clarify the behavior of coherent systems and/or positive (failure time) random variables. Among these, the following are worth special mention.

(i) Samaniego [11] derives a representation for the failure rate of an arbitrary coherent system when the lifetimes of its components are independently distributed according to a common absolutely continuous distribution F. The system failure rate is written explicitly as a function of F and its failure rate. The representation is used in several examples, including an example showing that the well-known closure theorem for k-out-of-n systems in i.i.d. IFR components proven by Barlow & Proschan can be extended, but not to all coherent systems. The class of coherent systems for which such closure obtains is characterized. ∎

(ii) Samaniego [13] discusses various methods and criteria for comparing coherent systems. A theoretical result is derived for comparing systems of a given order when components are assumed to have independent and identically distributed lifetimes. The result gives sufficient conditions for the lifetime of one system to be stochastically larger than that of another system. ∎

(iii) The identity $E(X) = \int_0^\infty \bar{F}(x)dx$, where X is a nonnegative random variable and $\bar{F}(x) = P(X>x)$ is well known in reliability theory and survival analysis. In a 1982 paper, Samaniego extended this identity to produce an expression of the k^{th} moment of a nonnegative random variable in terms of an integral involving the survival function:

$$E(X^k) = k!\int_0^\infty \cdots \int_0^\infty \bar{F}(x_1+\ldots+x_k)dx_1\ldots dx_k,$$

when the expectation exists. This identity can be used in establishing certain inequalities for the New Better Than Used (NBU) class of distributions. Arnold, Reneau and Samaniego [1] generalize the identity above or arbitrary multivariate random variables. They then use the result to derive inequalities for multivariate extensions of the family of NBU distributions.

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